Landau theory of 2nd order phase transitions

Phase transitions

I order

Latent heat absorbed / released

$$Q = T(S_2 - S_1) = -T \frac{\partial F}{\partial T}\Big|_1^2$$

(Gas - liquid)

II order

$S_1 = S_2$

$\frac{\partial F}{\partial T}$ continuous

$\left(\frac{\partial^2 F}{\partial T^2}\right.$ discontinuous$\left.\right)$

Superconductive transition, paramagnet - ferromagnet

// Sometimes people talk about infinite - order phase transitions

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Focus on 2nd order phase transitions

Occur without a rapid transformation of the phase $\longrightarrow$ A continuous phase transition

It is accompanied by the change of the symmetry of the system though (spontaneous)

Example 1: ferromagnet - paramagnet transition between $\vec{M} \neq 0$ and $\vec{M} = 0$

Example 2: transition between a superconductor and a normal metal

Example 3: atoms shift in a solid

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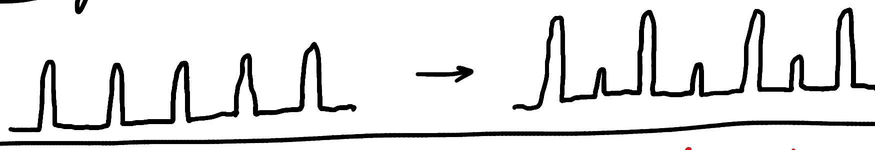


Ferromagnet – paramagnet phase transition

$M \neq 0$	$M = 0$
Ferromagnet	Paramagnet

// Assume a given volume and temperature

$$F = F_0 + \vec{\alpha}\vec{M} + AM^2 + \vec{C}\vec{M}^3 + BM^4 + \dots$$

near the transition point.

we are talking here about a system at constant temperature T and volume V.
For fixed T and P we would consider the Gibbs thermodynamic potential φ

One should have $\alpha = 0$. (also by symmetry)

Coeff. $A = A(V, T)$ depends on the volume and the temperature. Otherwise there will always be a local maximum or a minimum across the transition

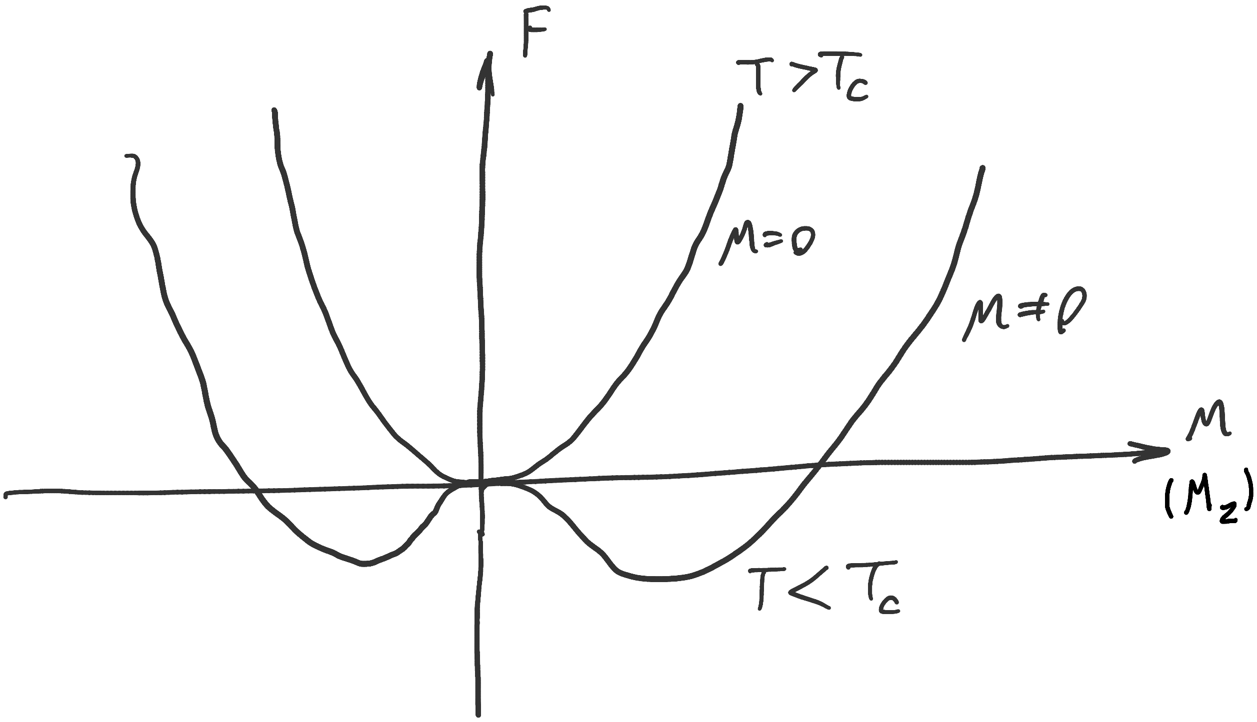
$$F \approx F_0 + AM^2$$

At the transition it changes sign

At the transition it changes [illegible]

$A = a(T - T_c)$

$F = F_0 + a(T - T_c)M^2 + BM^4$



Consider the „ordered" phase with $M \neq 0$

It corresponds to $T < T_c$

$2BM^2 - a(T_c - T) = 0$

$$\rightarrow M = \pm \left(\frac{a(T_c - T)}{2B} \right)^{\frac{1}{2}}$$

Note: the magnetisation spontaneously breaks symmetry

Instead of the free energy we would use [illegible] $\Phi(P, T)$ if the

Instead of the free energy we minimize the thermodynamic potential $\Phi(P,T)$ if the system was under constant pressure P and T.

The entropy

$$S = -\frac{\partial F}{\partial T} = S_0 - aM^2 - a(T-T_c)\frac{\partial M^2}{\partial T}$$

Use $M^2 = \frac{a(T_c - T)}{2B}$ below the transition

$$S = \begin{cases} S_0 , & T > T_c \\ S_0 + \frac{a^2(T-T_c)}{B} , & T < T_c \end{cases}$$

Heat capacity: (near the transition)

$$C = \begin{cases} C_0 , & T > T_c \\ C_0 + \frac{a^2 T_c}{B} , & T < T_c \end{cases}$$

Jump in heat capacity is a generic feature of (2nd order) phase transitions

Note: $\frac{\partial^2 F}{\partial T^2}$ is discontinuous

C_P, C_V and a lot of other quantities are discontinuous

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apply some external field H

Let's apply some external field H

$$F = F_0 + a(T-T_c)M^2 + BM^4 - HM$$

Note: an arbitrarily small field H will lead to a finite magnetisation M

Minimisation:

$$2aM(T-T_c) + 4BM^3 - H = 0$$

In the limit $H \to 0$ in the disordered phase neglect the $\propto M^3$ term

$$2aM(T-T_c) - H = 0$$

$$\chi = \frac{M}{H} = \frac{1}{2a(T-T_c)}$$

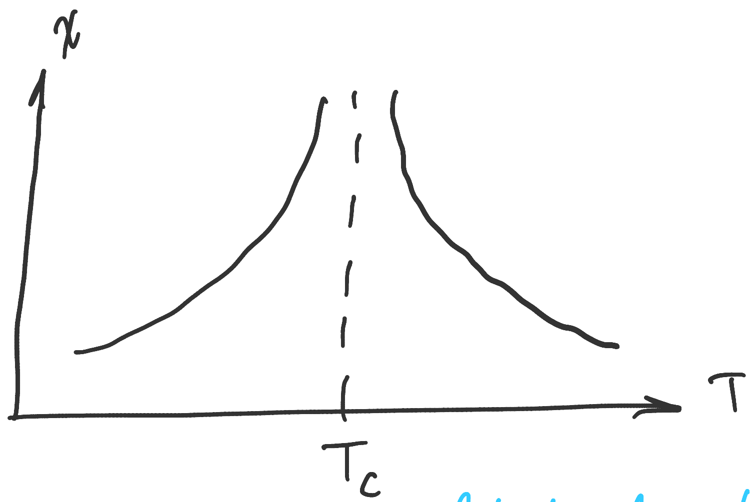
$$\chi \propto \frac{1}{T-T_c}$$ - Curie-Weiss law

Below the transition

$$\frac{dM}{dH}\left[2a(T-T_c) + 12B\frac{a(T_c-T)}{2B}\right] = 1$$

$$\frac{dM}{dH} \cdot 4a(T_c-T) = 1$$

$$\chi = \frac{dM}{dH} = \frac{1}{4a(T_c - T)}$$



(Also a way to detect phase transitions)

Inhomogeneos systems

We would need to introduce the density of the free energy

$$F = F_0 + \int f \, d\vec{r},$$

where f has the same form as the Ginzburg-Landau functional we considered earlier + one more term which accounts for inhomogeneities

$$F = F_0 + \frac{1}{2}\int \left[a(T-T_c) M^2 + a T_c \xi_0^2 (\vec{\nabla} M)^2 + \frac{B}{4} M^4 \right] d\vec{r}$$

That defines the characteristic length scale

$$\boxed{\xi^2 = \xi_0^2 \frac{T_c}{T - T_c}}$$

the correlation length

of the fluctuations above the transition.

of the fluctuations above the transition.

We may Fourier-transform the order parameter

$$\vec{M}(\vec{r}) = \int \frac{d\mathbf{k}}{(2\pi)^d} e^{i\mathbf{k}\vec{r}} \vec{M}_{\mathbf{k}} \qquad (M_k = M_{-k}^*)$$

$$F \approx F_0 + \frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^d} |M_{\mathbf{k}}|^2 a(T-T_c)(1+\xi^2 k^2) \equiv$$

$$\equiv \frac{1}{2}\frac{1}{V}\sum_{\mathbf{k}} |M_{\mathbf{k}}|^2 a(T-T_c)(1+\xi^2 k^2) + F_0$$

The partition function

$$Z = Z_0 \int \mathcal{D} M_k \, e^{-\frac{1}{2VT}\sum_k |M_k|^2 a(T-T_c)(1+\xi^2 k^2)}$$

$$F = F_0 - T\sum_{\mathbf{k}} \frac{1}{2} \ln \frac{\pi V T}{a(T-T_c)(1+\xi^2 k^2)}$$

We have to differentiate this wrt temperature T.

Usually people care about the most singular contribution and differentiate only the term $a(T-T_c)$ under the ln

$$\text{So, } S \to T\frac{\partial \ln Z}{\partial T}$$

$$C \to T^2 \frac{\partial^2 \ln Z}{\partial T^2} =$$

(that's a correction to what was there without fluctuations)

$$C \to T \frac{\quad}{\partial T^2} -$$ was there without fluctuations)

$$= -\frac{1}{2} T^2 \sum_k \frac{\partial^2}{\partial T^2} \ln\left[a(T-T_c) + \xi_0^2 k^2\right]$$

$$= \frac{T^2}{2} \sum_k \frac{a^2}{\left[a(T-T_c) + \xi_0^2 k^2\right]^2} \equiv V \frac{a^2 T^2}{2} \int_k \frac{1}{(a(T-T_c) + \xi_0^2 k^2)^2}$$

That is UV-divergent in dimensions $d > 4$ and is convergent (UV) otherwise

The behaviour of the fluctuational correction just adds a constant to the heat capacity. Otherwise, the heat capacity is singular in temperature T ($d_c = 4$ – upper critical dimension)

In 3D

$$\delta C = \frac{1}{2} V \frac{T^2}{(T-T_c)^2} \int \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{(1+\xi^2 k^2)^2} =$$

$$= \frac{1}{2} \cdot 4\pi \cdot \frac{\pi}{4} \frac{1}{(2\pi)^3} \left(\frac{T_c}{T-T_c}\right)^2 V \xi^{-3} =$$

$$= \frac{V \xi_0^{-3}}{16\pi} \left(\frac{T_c}{T-T_c}\right)^{\frac{1}{2}}$$

$$\boxed{\delta C = \frac{V \xi_0^{-3}}{16\pi} \left(\frac{T_c}{T-T_c}\right)^{\frac{1}{2}}}$$

Singular when $T \to T_c$!

... the fluctuational correction to be

Requiring the fluctuational correction to be larger than

$$\frac{a^2}{B}(T-T_c) \gg \xi_0^{-3}\left(\frac{T_c}{T-T_c}\right)^{\frac{1}{2}} \rightarrow$$

$$\rightarrow \frac{T-T_c}{T_c} \gg \underbrace{\left(\frac{B}{a^2\,\xi_0^3\,T_c}\right)}_{\text{Gi}}{}^{\frac{3}{2}}$$ – Ginzburg criterion